Deformations and electromagnetic moments of light exotic nuclei

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Abstract. Structure of carbon and neon isotopes is investigated by deformed Skyrme Hartree-Fock and shell model calculations. We point out that the quadrupole deformations of C and Ne isotopes have strong isotope dependence as a typical example of the evolution of deformations in nuclei. It is shown also that the quadrupole moments and the magnetic moments of the odd C isotopes depend clearly on assigned configurations, and their experimental data will be useful to determine the spin-parities and the deformations of the ground states of these nuclei. The electric quadrupole (E2) transitions in even C and Ne isotopes are also studied. The isotope dependence of the $E2$ transition strength is reproduced properly, although the calculated strength overestimates an extremely small observed value in ${}^{16}C$.

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1 Introduction

Nuclei far from the stability lines open a new test ground for nuclear models. Recently, many experimental and theoretical efforts have been paid to study the structure and reaction mechanism in nuclei near drip lines. Modern radioactive nuclear beams and experimental detectors reveal several unexpected structure of light nuclei with the mass number $A \sim (10-24)$ such as existence of halo and skin [\[1\]](#page-3-0), modifications of shell closures [\[2\]](#page-3-1) and Pigmy resonances in electric dipole transitions [\[3\]](#page-3-2). One of the current topics is a large quenching of the electric quadrupole $(E2)$ transition between the first excited 2^+ state and the ground state in ${}^{16}C$ [\[4,](#page-3-3)[5\]](#page-3-4).

The isotope dependence of deformation is an interesting subject to study in relation to the evolution of deformation in quantum many-body systems as a manifestation of spontaneous symmetry breaking effect. To this end, C and Ne isotopes are promising candidates since all isotopes between the proton and neutron drip lines will be available in future experiments within next few years. The effect of spontaneous symmetry breaking effect is a general phenomenon known in many fields of physics. In molecular physics, the spontaneous symmetry breaking was discovered by Jahn and Teller in 1937 [\[6\]](#page-3-5). The coupling to the quadrupole vibration is the main origin of the static deformation in both molecules and atomic nuclei [\[7\]](#page-3-6). On the other hand, the pairing correlations in nuclei work to stabilize the spherical symmetry. A unique and essential feature of the evolution of deformation in atomic nuclei will appear in the competition between the deformation deriving particle-vibration coupling and the pairing correlations [\[8\]](#page-3-7).

2 Deformations of carbon and neon isotopes

We investigate the neutron number dependence of deformations along the chains of C and Ne isotopes. For this purpose, we perform deformed HF+BCS calculations with a Skyrme interaction SGII. The axial symmetry is assumed for the deformed HF potential. The pairing interaction is taken to be a density-dependent pairing interaction in the BCS approximation;

$$
V(\mathbf{r}_1, \mathbf{r}_2) = V_0' \left(1 - \frac{\rho(r)}{\rho_0}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2),\tag{1}
$$

where $\rho(r)$ is the HF density at $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and ρ_0 is chosen to be $0.16 \,\mathrm{fm}^{-3}$. The pairing strength is taken to be $-410\,\mathrm{MeV}\cdot\mathrm{fm}^3$ for both neutrons and protons. A smooth energy cut-off is employed in the BCS calculations [\[9\]](#page-3-8).

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Fig. 1. Deformed $HF + BCS$ calculations with SGII interaction; (a) for C isotopes and (b) for Ne isotopes. The densitydependent pairing interaction [\(1\)](#page-0-0) is adopted in the calculations. The strength of spin-orbit force is modified to be 60% of the original one in the HF calculations of C isotopes. See the text for details.

Figure [1](#page-1-0) shows the binding energy surfaces for evenmass C and Ne isotopes as a function of the quadrupole deformation parameter β_2 with SGII interaction. The spin-orbit interaction of SGII interaction is reduced to be 60% of the original strength to enhance the deformation effect $[10]$. The energy minimum in ¹²C appears at oblate deformation with $\beta_2 = -0.32$. The energy minimum becomes spherical in ¹⁴C because of the neutron closed shell effect. For heavier C isotopes 16 C and 18 C. two minima appear both in the prolate and oblate sides. In ¹⁸C, the ground state has the largest deformation at $\beta_2 = 0.36$, while the local minimum appears at the oblate side with $\beta_2 \sim -0.3$. The deformations become oblate in 20° C and 2° C. The HF calculation with the original spinorbit strength gives a spherical shape for 22 C.

The deformations of Ne isotopes are shown in fig. $1(b)$ $1(b)$. The similar neutron number dependence to that of C isotopes is found in Ne isotopes. In general, the energy surfaces are shallow in even Ne isotopes. The energy minimum of ¹⁸Ne is very close to the spherical shape as ex-

Fig. 2. Isotope dependence of deformations of C and Ne isotopes with SGII interaction. The points with error bars show the cases in which two deformation minima are found within the energy difference of 0.1 MeV.

pected due to the $N = 8$ shell closure. The prolate deformations are found in 20 Ne and 22 Ne and then the deformation minimum is oblate in ²⁴Ne. The spherical and oblate energy minima are almost degenerate in energy in ²⁶Ne. The energy minimum of 28 Ne is very flat and depends on the adopted interaction. Namely the SGII interaction gives a prolate minimum, while the SIII interaction gives an oblate minimum. The nucleus ³⁰Ne is found to show a spherical shape due to another shell closure $N = 20$. We can see also a prolate local minimum in the case of ³⁰Ne, whose binding energy is very close to that of the spherical minimum. Then, $32\degree$ Ne becomes prolate again as a typical nucleus next to the closed shell nucleus.

Figure [2](#page-1-1) shows the isotope dependence of the quadrupole deformation parameter β_2 at the binding energy minima for C and Ne isotopes. Blocked deformed Skyrme HF+BCS calculations are performed for odd carbon isotopes. The results show that the ground state of ¹⁷C is prolate with $J^{\pi} = \frac{3}{2}$ $^+$ while that of 19 C is oblate with $J^{\pi} = \frac{3}{2}$ ⁺. In ¹⁹C, the energy minimum of $J^{\pi} = \frac{1}{2}$ + is very close to that of $J^{\pi} = \frac{3}{2}$ + having almost the same oblate deformation $\beta_2 \sim -0.36$. These spin-parities of the ground states are consistent with our shell model calculations $[11]$. The spin of the ground state of ${}^{17}C$ was assigned as $3/2^+$ in the magnetic moment measurement [\[12\]](#page-3-11). The spin of ¹⁹C was assigned as $1/2^+$ in the Coulomb break-up reactions [\[13\]](#page-3-12), while there is still controversial argument on the experimental assignment in ref. [\[14\]](#page-3-13). In fig. [2,](#page-1-1) the two isotopes show a clear manifestation of the evolution of the nuclear deformation induced by the deformation driving force in the mean field potential [\[8\]](#page-3-7). The deformation effect in the C and Ne isotopes is unique compared with that in rare-earth nuclei in a sense that both prolate and oblate deformations appear clearly in the beginning of the closed shell and at the end of the closed shell.

80

3 Q-moments, μ -moments and E2 transitions

We study electromagnetic moments and transitions in C and Ne isotopes by the shell model calculations [\[15\]](#page-3-14) in comparison with the deformed HF results [\[10\]](#page-3-9). We first discuss the Q-moments. The WBP interaction is used as an effective interaction within the $0\hbar\omega$ space. We adopt state-dependent polarization charges which are obtained by the microscopic particle vibration coupling model (Hartree-Fock + Random Phase Approximation $[16]$). These polarization charges can be parameterized as

$$
\frac{e_{\text{pol}}}{e} = a\frac{Z}{A} + b\frac{N-Z}{A} + \left(c + d\frac{Z}{A}\frac{N-Z}{A}\right)\tau_z \tag{2}
$$

with $a = 0.82$, $b = -0.25$, $c = 0.12$ and $d = -0.36$ to reproduce the calculated values for 12 C and 16 C in ref. [\[16\]](#page-3-15). Both the neutron (ν) and proton (π) polarization charges decrease as the neutron excess increases. The effective charges are given by

$$
e_{\text{eff}} = e(1/2 - t_z) + e_{\text{pol}}.
$$
 (3)

The Q-moments obtained by using these polarization charges are shown in fig. $3(a)$ $3(a)$. Open circles denote results of the shell model calculations with the use of e_{pol} . Single-particle or -hole values with the use of e_{pol} are given by open triangles. The configurations for ${}^{9}C$ and ${}^{11}C$ are $\nu p_{3/2}$ and $\nu p_{3/2}^{-1}$, respectively. The configurations for ¹⁷C and ¹⁹C are $\nu d_{5/2} 1s_{1/2}^2$ and $\nu d_{5/2}^{-1} (\nu d_{5/2}^5)$, respectively, for the $5/2^+$ state. For the $3/2^+$ state of ¹⁷C, a case for a $+\text{ state}$ For the $2/9^+$ single particle configuration of $\nu d_{3/2}$ is given.

Filled triangles are obtained for $\nu d_{5/2}^{\pm 2} 1s_{1/2}$ configuration with the use of e_{pol} . The $\nu d_{5/2}^2(J=2)1s_{1/2}$ and $\nu d_{5/2}^{-2}(J=2)1s_{1/2}$ are possible simple configurations for ¹⁷C and ¹⁹C, respectively, since the $\nu d_{5/2}^3$ or $\nu d_{5/2}^3 1s_{1/2}^2$ configuration corresponding to the middle of the $d_{5/2}$ shell results in the vanishing of the Q-moments. The Q-moments are given by $\mp \frac{2}{5} e_{\text{pol}}^n Q_{\text{sp}}(d_{5/2})$ for $3/2^+$ and $\mp \frac{4}{7} e_{\text{pol}}^n Q_{\text{sp}}(d_{5/2})$ for $5/2^+$ in the case of the $\nu d_{5/2}^{\pm 2}(J =$ $2)1s_{1/2}$ configuration. Here, e_{pol}^n is the neutron polarization charge and $Q_{\rm sp}(d_{5/2})$ is the single particle value of the Q -moment for $d_{5/2}$. Note that the signs of the Q -moments for ${}^{17}C$ and ${}^{19}C$ are opposite.

The shell model values of the Q-moments are obtained by the admixture among these configurations, and their magnitudes are usually enhanced compared to those of the simple configurations. Nevertheless, the difference of the signs between ${}^{17}C$ and ${}^{19}C$ can be understood from those of the simple configurations.

Calculated values for the magnetic (μ) moments are shown in fig. [3\(](#page-2-0)b). Here, $g_s^{\text{eff}}/g_s^{\text{free}} = 0.9$ is used for neutron. The values of the μ -moments are found to be sensitive to the configurations as in the case for the Q-moments, which is useful to find out the spin-parities and the deformations of the ground states of these nuclei. Especially, the spin assignments of ${}^{17}C$ and ${}^{19}C$ are quite consistent between the deformed HF and the shell model

Fig. 3. Q-moments and magnetic moments for the odd C isotopes. (a) Open triangles denote Q-moments of single-particle or -hole values, while filled triangles give the results of the $0d_{5/2}^{\pm 2}1s_{1/2}$ configuration. These values include the effects of the polarization charges, e_{pol} in eq. [\(2\)](#page-2-1). (b) Open circles denote the results of the magnetic $g\text{-factors}$ of shell model calculations obtained with the use of WBP interaction and $g_s^{\text{eff}} = 0.9g_s$, while the HF results are obtained by using deformed HF wave functions and shown by open boxes. The filled circles are the experimental values taken from refs. [\[17,](#page-3-16)[18,](#page-3-17)[19,](#page-3-18)[12,](#page-3-11)[20\]](#page-3-19).

results. The $5/2^+$ state is excluded from the ground state by the β -decay experiment.

Let us now discuss the E2 transitions in the even C and Ne isotopes. Calculated and experimental $B(E2)$ values for the $2^+_1 \rightarrow 0^+_{\text{g.s.}}$ transitions are shown in fig. [4.](#page-3-20) The shell model values obtained with the use of e_{pol} are larger than the experimental values except for ${}^{10}C$, for which larger effective charges of eqs. [\(2\)](#page-2-1) and [\(3\)](#page-2-2) $e_{\text{eff}}^p = 1.38$ and $e_{\text{eff}}^{n} = 0.71$ are needed. For 12^{-16} C, the isotope dependence of the observed values $[20,4]$ $[20,4]$ is well explained by that of e_{pol} , but their magnitudes are smaller than the calculations. In particular, the observed $B(E2)$ value is quite small for ${}^{16}C-[4]$ ${}^{16}C-[4]$, which suggests some exotic structure yet unknown in the isotopes, for example, the shape coexistence of prolate and oblate deformations expected

Fig. 4. $B(E2)$ values of the $2^+_1 \rightarrow 0_{\text{g.s.}}$ transitions; (a) C isotopes and (b) Ne isotopes. Filled and open squares show the results of the shell model calculations with the use of the constant effective charges $e_p^{\text{eff}} = 1.3$, $e_n^{\text{eff}} = 0.5$ and the harmonicoscillator wave functions with $b = 1.64$ fm for C isotopes and $b = 1.83$ fm for Ne isotopes. The WBP and MK interactions are used to calculate the shell model wave functions for the filled and open squares, respectively. Filled (WBP) and open (MK) diamonds are obtained with the use of HF wave functions and the isotope-dependent polarization charges e_{pol} given by eq. [\(2\)](#page-2-1). Filled circles show experimental values [\[20,](#page-3-19)[4\]](#page-3-3).

from the deformed HF calculations. It would be also interesting to find out if the $B(E2)$ value increases for ¹⁸C as the calculation predicts. This increase comes from that of the neutron contribution.

4 Summary

We have studied the isotope dependence of deformation in C and Ne isotopes by using the deformed HF calculations with BCS approximation. We found clear isotope dependence of the deformation change as a manifestation of the dynamical evolution of nuclear deformation. The configuration dependence of the Q-moments and μ -moments in the odd C isotopes, which can be attributed to the deformation effects, is also pointed out by using the shell model wave functions. This dependence can be used to determine the spin-parities as well as the deformation properties of the ground states of the isotopes. The isotope dependence of the $B(E2)$ values in even C and Ne isotopes is reproduced well by the calculations, while the experimental values are found to be smaller in 12^{-16} C, in particular, in ¹⁶C where the observed $B(E2)$ value almost vanishes. This suggests an exotic structure of ¹⁶C still to be found out.

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References

- 1. I. Tanihata et al., Phys. Rev. Lett. 55, 2676 (1985).
- 2. A. Ozawa et al., Phys. Rev. Lett. 84, 5493 (2000).
- 3. A. Leistenschneider et al., Phys. Rev. Lett. 86, 5442 (2001).
- 4. N. Imai et al., Phys. Rev. Lett. 92, 062501 (2004).
- 5. Z. Elekes et al., Phys. Lett. B 586, 34 (2004).
- 6. H.A. Jahn, E. Teller, Proc. R. Soc. London, Ser. A 161, 220 (1937).
- 7. P.-G. Reinhard, E.W. Otten, Nucl. Phys. A 420, 173 (1984)
- 8. W. Nazarewicz, Int. J. Mod. Phys. E 2, 51 (1993); Nucl. Phys. A 574, 27c (1994).
- 9. M. Bender, K. Rutz, P.-G. Reinhard, J.A. Maruhn, Eur. Phys. J. A 8, 59 (2000); P.-G. Reinhard, the computer code SKYAX, unpublished.
- 10. H. Sagawa, X.R. Zhou, X.Z. Zhang, T. Suzuki, to be published in Phys. Rev. C.
- 11. T. Suzuki, H. Sagawa, K. Hagino, in the Proceedings of the International Symposium on "Frontiers of Collective Motions (CM2002)". (World Scientific, 2003) p. 236; H. Sagawa, T. Suzuki, K. Hagino, Nucl. Phys. A 722, 183 (2003).
- 12. H. Ogawa et al., Eur. Phys. J. A 13, 81 (2002).
- 13. D. Bazin et al., Phys. Rev. C 57, 2156 (1998); T. Nakamura et al., Phys. Rev. Lett. 83, 1112 (1999); V. Maddalena et al., Phys. Rev. C 63, 024613 (2001).
- 14. Rituparna Kanungo, I. Tanihata, Y. Ogawa, H. Toki, A. Ozawa, Nucl. Phys. A 677, 171 (2000).
- 15. E.K. Warburton, B.A. Brown, Phys. Rev. C 46, 923 (1992); OXBASH, the Oxford, Buenos-Aires, Michigan State, Shell Model Program; B.A. Brown et al., MSU Cyclotron Laboratory Report No. 524, 1986.
- 16. H. Sagawa, K. Asahi, Phys. Rev. C 63, 064310 (2001).
- 17. K. Matsuta et al., Nucl. Phys. A 588, 153c (1995).
- 18. P. Raghaven, At. Data Nucl. Data Tables 42, 189 (1989).
- 19. K. Asahi et al., AIP Conf. Proc. 570, 109 (2001).
- 20. S. Raman et al., At. Data Nucl. Data Tables 36, 1 (1987).